



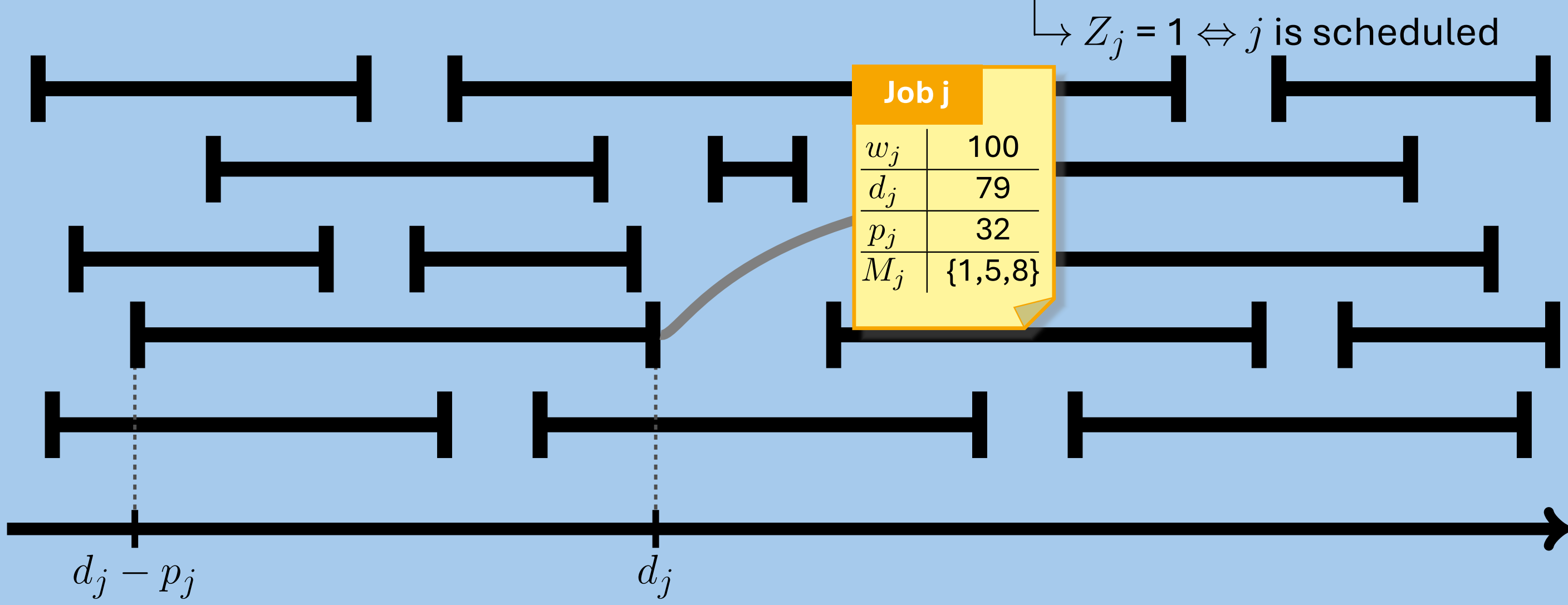
Interval Scheduling with Eligible Machines

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Interval Scheduling with Eligible Machines

Input: A set of n jobs $\mathcal{J} = \{1, \dots, n\}$, a set of m machines, and an integer W^* . Each job $j \in \mathcal{J}$ has a processing time $p_j \in \mathbb{N}$, a due date $d_j \in \mathbb{N}$, a weight $w_j \in \mathbb{N}$, and a set of eligible machines $M_j \subseteq \{1, \dots, m\}$.

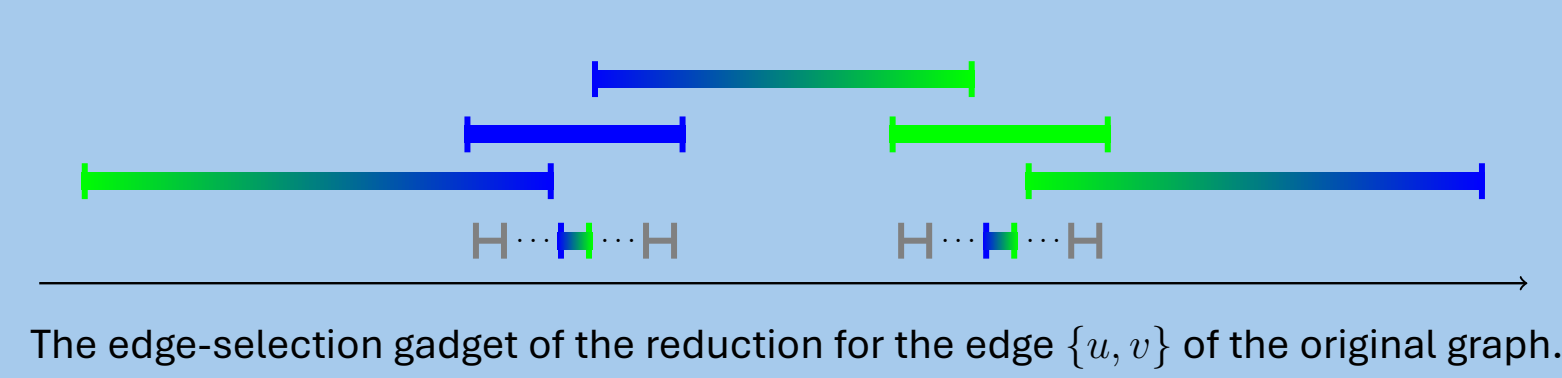
Question: Is there a feasible schedule for \mathcal{J} such that $\sum w_j Z_j \geq W^*$?



Our Contribution

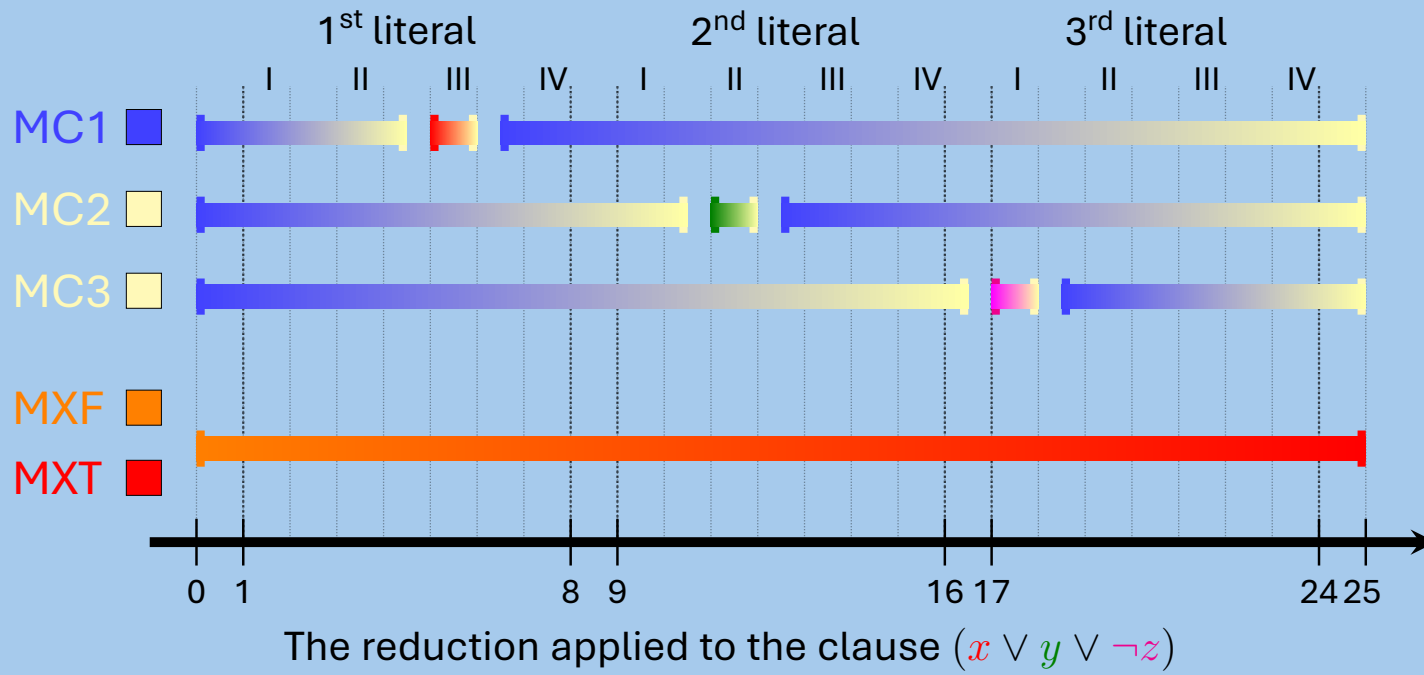
Theorem 1. Interval Scheduling with Eligible Machines is strongly $W[1]$ -hard w.r.t. m .

- Reduction from Multicolored Clique: Simulating edges with job-gadgets.



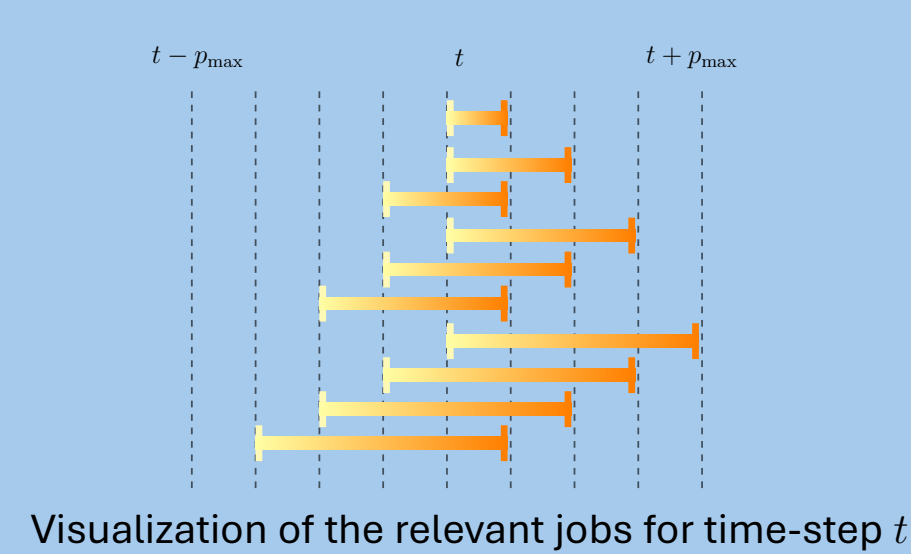
Theorem 2. Interval Scheduling with Eligible Machines NP-hard for constant p_{\max} .

- Reduction from Exact (3-4)-SAT:



Theorem 3. Interval Scheduling with Eligible Machines is FPT w.r.t. $m + p_{\max}$.

- Data reduction: Deleting irrelevant jobs. Retaining top-weighted jobs for each timestep t .
- $O(m^2 p_{\max}^2)$ jobs remain per t .
- Dynamic program on the time-steps solves the problem in $O((m^2 p_{\max}^2)^{2m} nm)$ -time.



Related Work

Arkin & Silverberg

Scheduling Jobs with Fixed Start and End Times 1987

- Proved NP-hardness.
- Gave an $O(n^{m+1})$ -time algorithm

Mnich & van Bevern

Parameterized Complexity of Machine Scheduling: 2018
15 Open Problems

- Posed **Open Question 8:** Is the Problem FPT w.r.t. m ?

Hermelin, Itzhaki, Molter, Shabtay

On the Parameterized Complexity of Interval Scheduling with Eligible Machine Sets 2024

- Answered Open Question 8 negatively.
- Left open whether an $(n^{O(\sqrt{m})})$ -time algorithm exists.

Classical Complexity

Studies how running time depends on input size n :

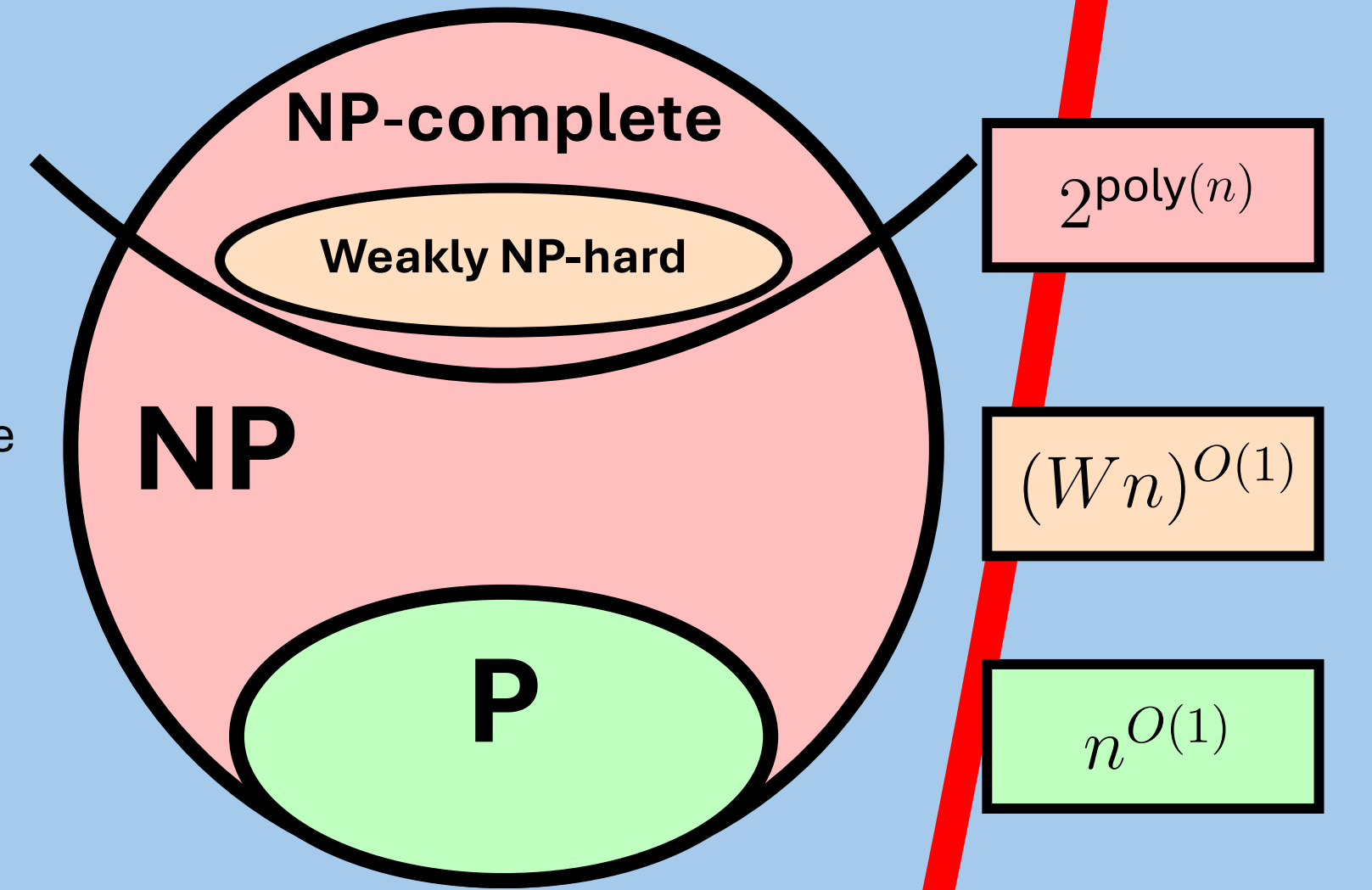
- Class P contains problems solvable in time $n^{O(1)}$.

- A problem is NP-hard if every problem in NP can be reduced to it. Such problems are solvable in exponential time $2^{\text{poly}(n)}$

and are believed not to admit polynomial-time algorithms.

- A problem is weakly NP-hard if its hardness arises from large numerical values in the input. Such problems admit pseudo-polynomial-time algorithms, whose runtime is $(Wn)^{O(1)}$

by which W is the largest numeric input value.



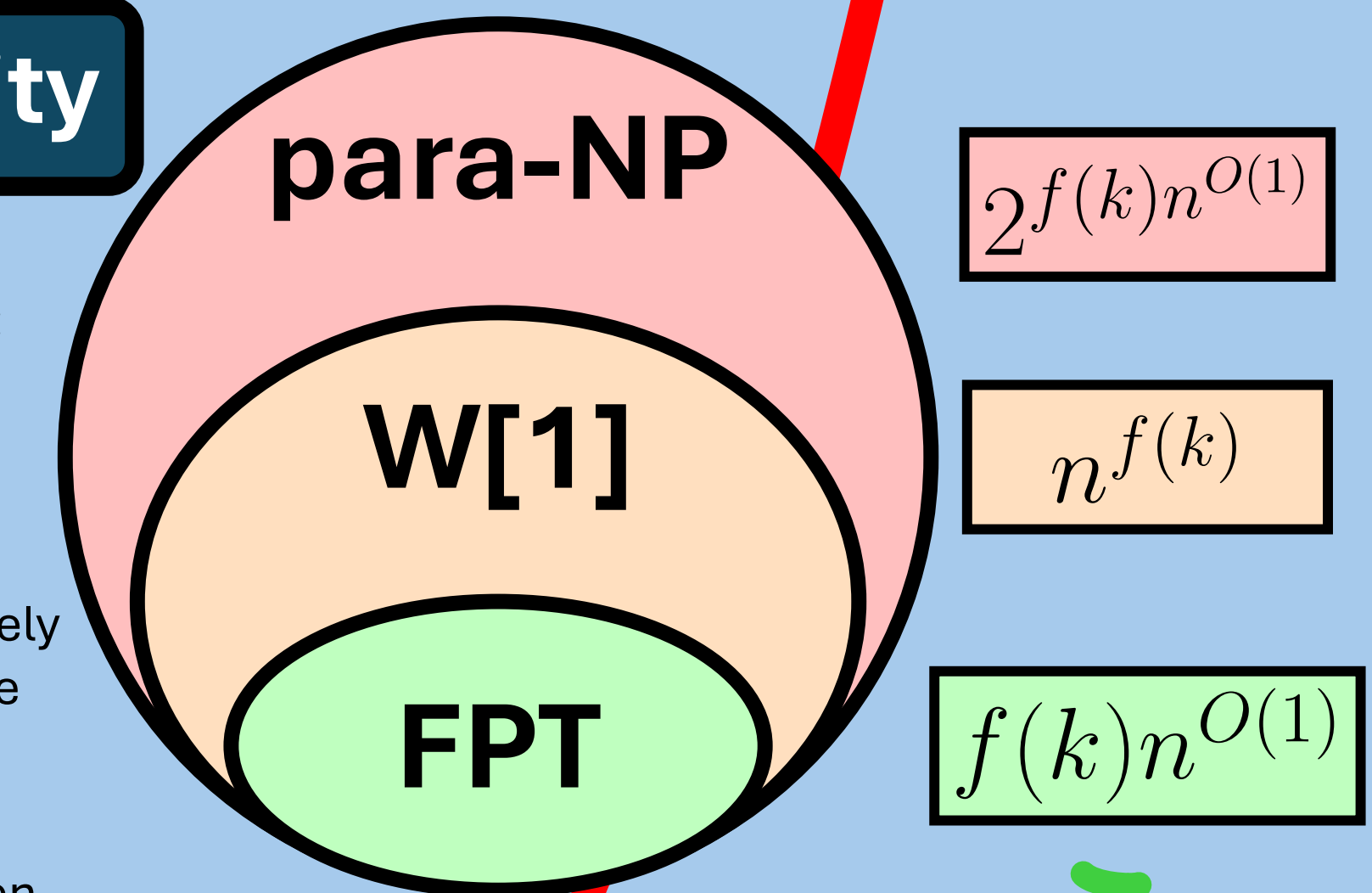
Parameterized Complexity

Studies the effect of input parameters on running time:

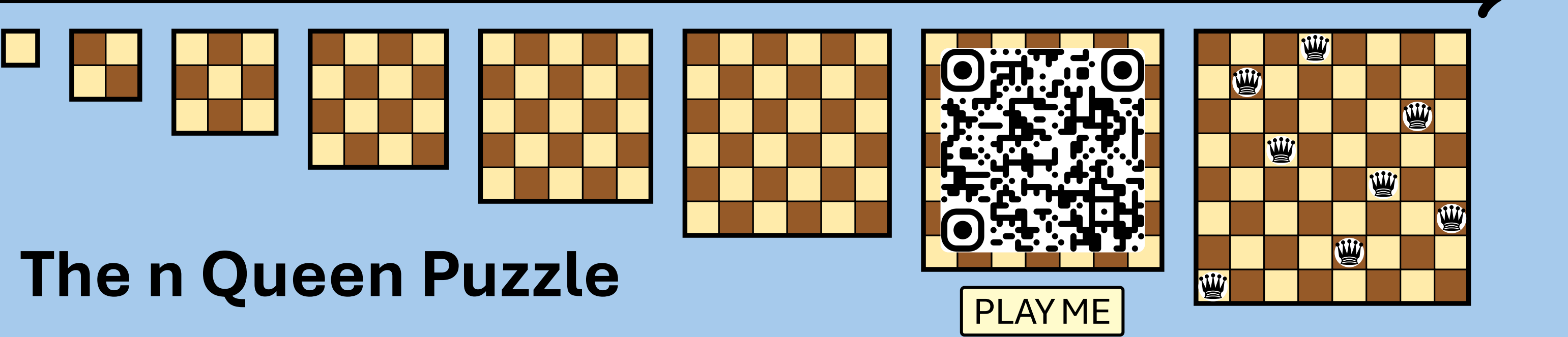
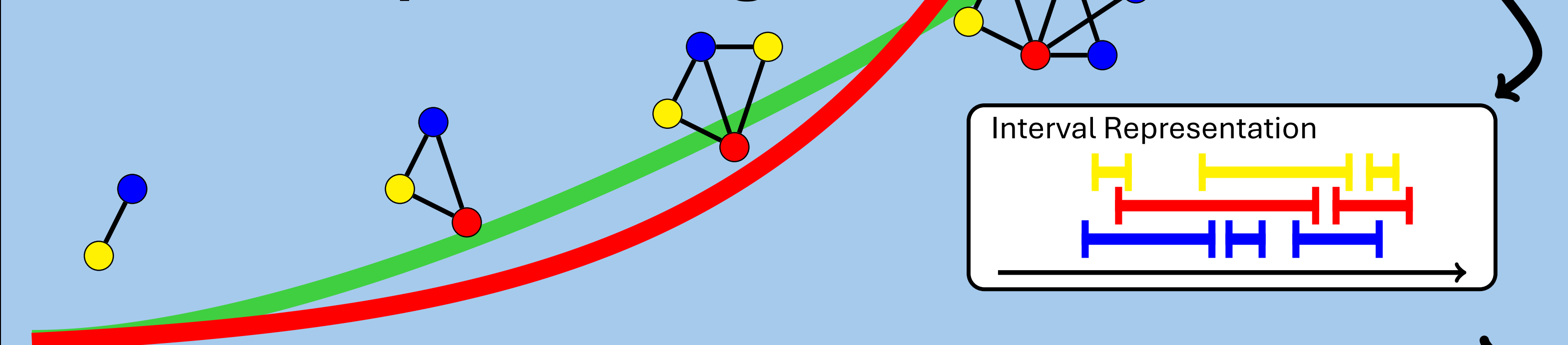
- A problem is fixed-parameter tractable (FPT) with respect to a parameter k if it can be solved in time $f(k) \cdot n^{O(1)}$.

- If a problem is $W[1]$ -hard with respect to k , it is unlikely to admit an FPT algorithm, the best we typically hope for is running time $n^{f(k)}$.

- If a problem is para-NP-hard, it remains NP-hard even for constant k , and hence cannot admit an $n^{f(k)}$ -time algorithm unless $P=NP$.

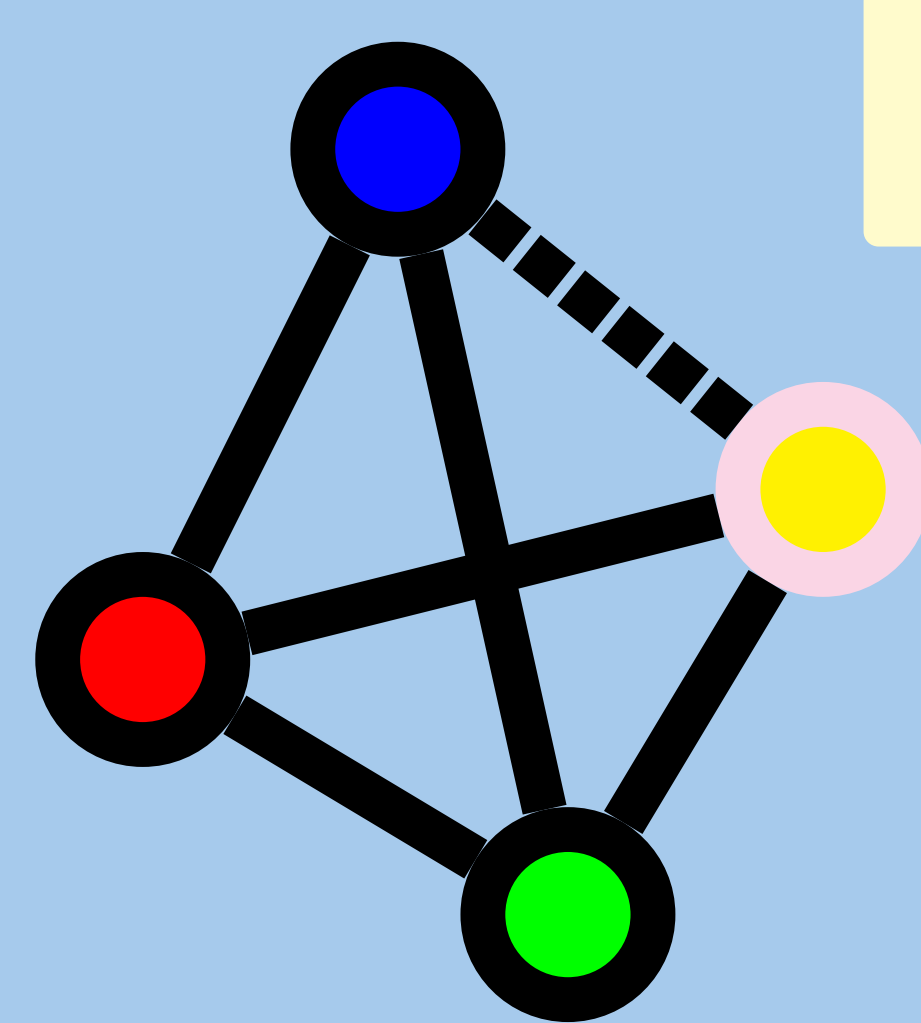


Interval Graph Coloring



Theorem 1. W[1]-hardness

Multicolored Clique



$\forall v \in V$ create VERTEX JOBS
 $\forall \{u, v\} \in E$ create an EDGE JOB

$$\binom{k}{2} + 1$$

#machines

Input: A graph G with a vertex-coloring and an integer k .
Question: Is there a k -sized clique with valid coloring?

Multicolored Clique is $W[1]$ -hard w.r.t. the clique size k [Fellows, Hermelin, Rosamond, Viallette 2009].

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